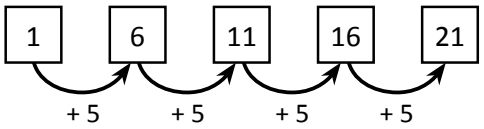
Patterns and functions – recursive number patterns

Look around you, can you see a pattern? A pattern is an arrangement of shapes, numbers or objects formed according to a rule. Patterns are everywhere, you can find them in nature, art, music and even in dance!

In this topic, we are looking at number patterns. A number pattern is a sequence or list of numbers that is formed according to a rule.

Number patterns can use any of the four operations (+, –, ×, ÷) or even a combination.

In the example below, if we follow this instruction: "starting at 1 add 5 each time" we get this number pattern:



1 **Write the next 3 numbers in each sequence by following the rule:**

a Rule: add 6 5 → 11 → 17 → ☐ → ☐ → ☐

b Rule: subtract 10 100 → 90 → 80 → ☐ → ☐ → ☐

c Rule: multiply by 2 2 → 4 → 8 → ☐ → ☐ → ☐

2 **Figure out the missing numbers in each pattern and write the rule. Circle the ascending patterns.**

a 14 21 ☐ 35 42 ☐

Rule _____

b 17 37 57 ☐ ☐ ☐

Rule _____

c 75 ☐ ☐ 30 15 ☐

Rule _____

d ☐ 16 24 ☐ 40 ☐

Rule _____

e 63 54 ☐ 36 27 ☐

Rule _____

f 63 56 ☐ 42 35 ☐

Rule _____

3 **Complete these grid patterns. Look closely at the numbers in the grid and follow the patterns.**

a

		32	
40		42	
50		52	

b

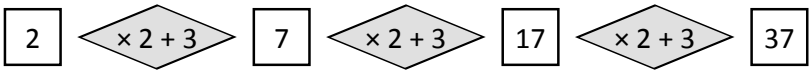
		66	
		76	
84			
		96	

c

3			
		17	
23	25		

Patterns and functions – recursive number patterns

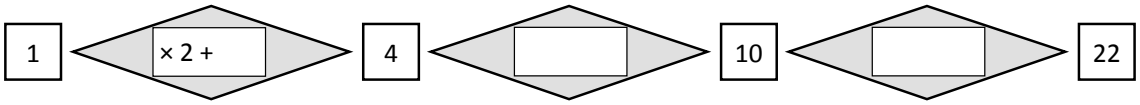
Some number patterns can be formed with 2 operations each time. For example:



The rule is multiply by 2 and add 3 each time.

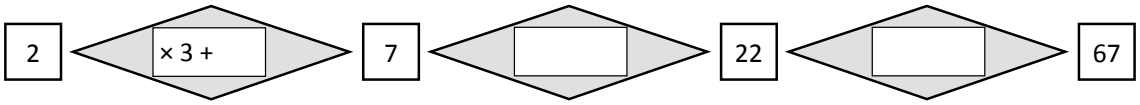
4 **With these number patterns, write the rule as 2 operations in the diamond shapes and describe it underneath.**

a



The rule is _____

b



The rule is _____

5 **Lena and Max were asked to show a number pattern for different rules. Check each sequence and put a circle around any errors. You may use a calculator.**

a Start at 2, add 1 and multiply by 2

Lena	2	6	14	30	62	126	254	510	1022

b Start at 3, add 1 and multiply by 2

Max	3	8	18	38	78	158	320	640	1280

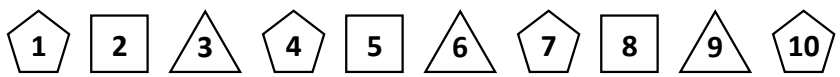
6 **Look at each pattern of shapes and see if you can predict the following:**

a

What will shape number 20 look like? Draw it here:

What will shape number 33 look like? Draw it here:

b



What will shape number 15 look like? Draw it here:

What will shape number 26 look like? Draw it here:

Patterns and functions – function number patterns

There are 2 different types of rules that a number pattern can be based upon:

1 A recursive rule – used to continue the sequence by doing something to the **number** before it.

2 A function rule – used to predict any number by applying the rule to the **position** of the number.

A function rule is a rule based on the position of a number.

Consider this. Lucia was given this number pattern: 5 10 15 20 25

Her teacher asked her to work out what the 20th number would be without continuing the sequence. Lucia used a table to work out the rule between the position of a number and the number in the pattern. She worked out the rule to be × 5.

Position of number	1	2	3	4	5	20
Function rule	× 5	× 5	× 5	× 5	× 5	× 5
Number pattern	5	10	15	20	25	100

So, following the rule based on the position of a number, the 20th number is 100.

This is a function rule.

1 **Use the function rule and then apply the rule to position 20.**

a

Position of number	1	2	3	4	5	20
Function rule						
Number pattern	6	12	18	24	30	

b

Position of number	1	2	3	4	5	20
Function rule						
Number pattern	4	8	12	16	20	

c

Position of number	1	2	3	4	5	20
Function rule						
Number pattern	8	16	24	32	40	

d

Position of number	1	2	3	4	5	20
Function rule	× 4 +					
Number pattern	7	11	15	19	23	

Patterns and functions – function number patterns

Function rules with 2 operations are easy to work out when we look at how they are linked to the multiplication tables.

Position of number	1	2	3	4	5
2 times table + 3	**2 + 3**	**4 + 3**	**6 + 3**	**8 + 3**	**10 + 3**
Number pattern	5	7	9	11	13
Function rule	Multiply by 2 and then add 3				

This table shows that the number pattern is the same as the 2 times table with 3 added to each answer.

2 **Complete each table to show how function rules with 2 operations can be linked to multiplication tables.**

a

Position of number	1	2	3	4	5
3 times table + _____	**3 + _____**	**6 + _____**	**9 + _____**	**12 + _____**	**15 + _____**
Number pattern	7	10	13	16	19
Function rule	Multiply by 3 and then add _____				

b

Position of number	1	2	3	4	5
6 times table + _____	**6 + _____**	**12 + _____**	**18 + _____**	**24 + _____**	**30 + _____**
Number pattern	8	14	20	26	32
Function rule	Multiply by 6 and then add _____				

c

Position of number	1	2	3	4	5
____ times table + ____					
Number pattern	11	19	27	35	43
Function rule					

3 **Complete this table to show the 4 times tables with 2 added.**

a

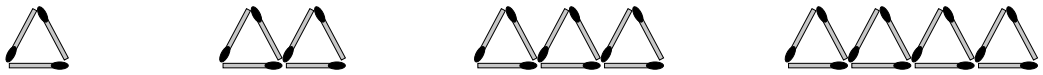
Position of number	1	2	3	4	5
4 times table + 2					
Number pattern					
Function rule					

b What would the number in the 20th position be? _____

Patterns and functions – matchstick patterns

Use the function rule to predict geometric patterns with matchsticks. Here is an example.
Mia made this sequence of shapes with matchsticks:

Shape 1 **Shape 2** **Shape 3** **Shape 4**



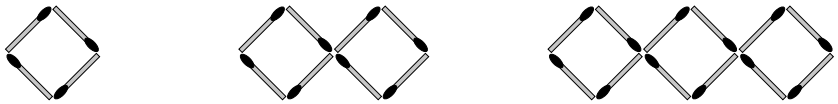
If Mia followed this sequence, how many matchsticks will she need for shape 20?

Shape number	1	2	3	4	5	**20**
Number of matchsticks	3	6	9	12	15	*60*
Function rule	Number of matchsticks = Shape number × *3*					

1 **Complete the table for each sequence of matchstick shapes. Use the function rule for finding the number of matchsticks needed for the shape in the 20th position.**

a

Shape 1 Shape 2 Shape 3

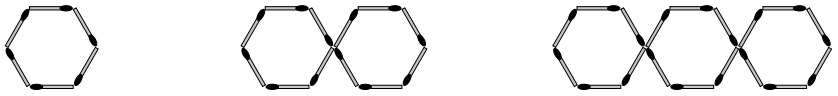


Shape number	1	2	3	4	5	**20**
Number of matchsticks	4	8	12			
Function rule	Number of matchsticks = Shape number × _____					

.....

b

Shape 1 Shape 2 Shape 3

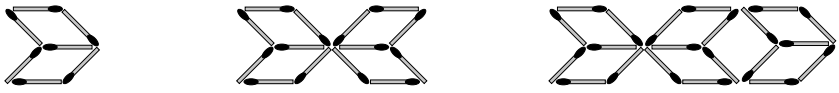


Shape number	1	2	3	4	5	**20**
Number of matchsticks	6	12	18			
Function rule	Number of matchsticks = Shape number × _____					

.....

c

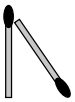
Shape 1 Shape 2 Shape 3

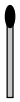


Shape number	1	2	3	4	5	**20**
Number of matchsticks	7	14	21			
Function rule	Number of matchsticks = Shape number × _____					

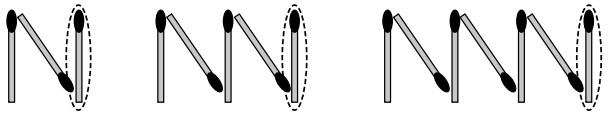
Patterns and functions – matchstick patterns

This time the rule for this matchstick pattern has 2 operations. Can you see why?
Look for a multiplication pattern and how many extra there are in each shape.

Look for a repeating element. 

Then look to see what is added.
These are circled in the sequence below. 

Shape 1 Shape 2 Shape 3

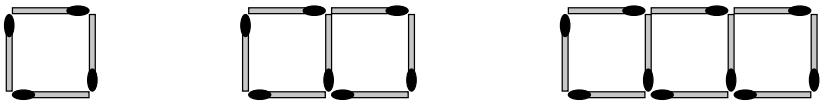


Shape 1 has 3 matchsticks $1 \times 2 + 1 = 3$
Shape 2 has 5 matchsticks $2 \times 2 + 1 = 5$
Shape 3 has 7 matchsticks $3 \times 2 + 1 = 7$

Shape number	1	2	3	4	5	20
Number of matchsticks	3	5	7	9	11	41
Function rule	Number of matchsticks = Shape number × 2 + 1					

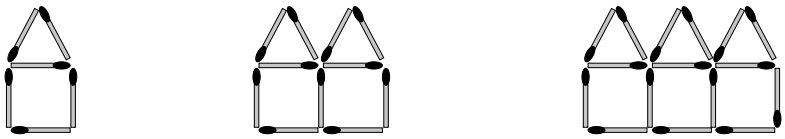
2 **In each of these patterns, look for the repeating element and then what is added each time:**

a Shape 1 Shape 2 Shape 3



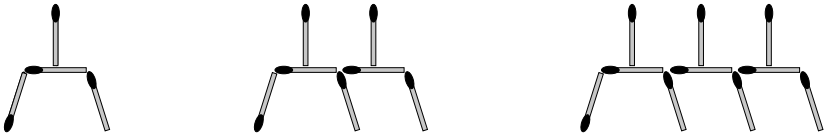
Shape number	1	2	3	4	5	20
Number of matchsticks	4	7	10			
Function rule	Number of matchsticks = Shape number × _____ + _____					

b Shape 1 Shape 2 Shape 3



Shape number	1	2	3	4	5	20
Number of matchsticks						
Function rule	Number of matchsticks = Shape number × _____ + _____					

c Shape 1 Shape 2 Shape 3

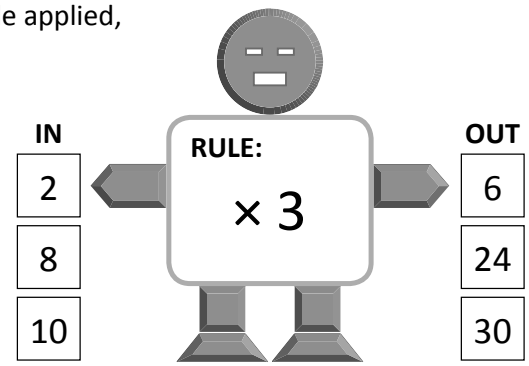


Shape number	1	2	3	4	5	20
Number of matchsticks						
Function rule	Number of matchsticks = Shape number × _____ + _____					

Patterns and functions – function machines

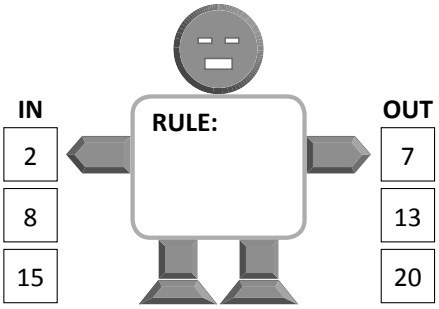
This is a function machine.

Numbers go in, have the rule applied, and come out again.

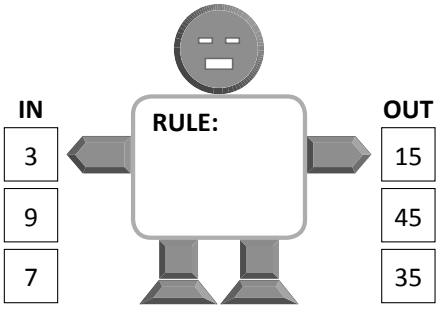


1 **Look carefully at the numbers going *in* these function machines and the numbers coming *out*. What rule are they following each time?**

a

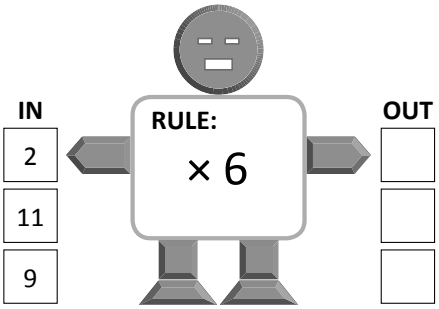


b



2 **What numbers will come *out* of these function machines?**

a



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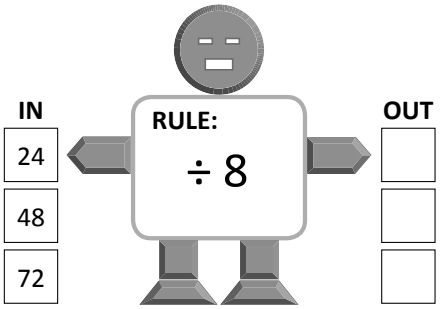
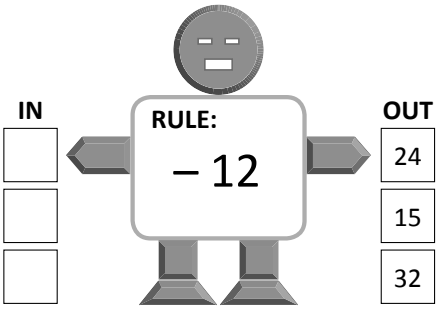


3 **What numbers go *in* to these number function machines?**

a



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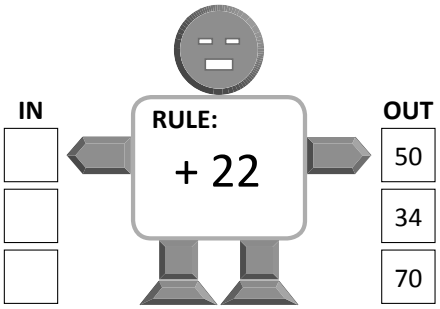
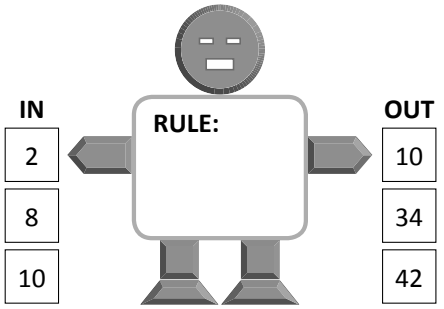


4 **Write the rule in each double function machine. Each rule is made up of 2 operations (× then +).**

a



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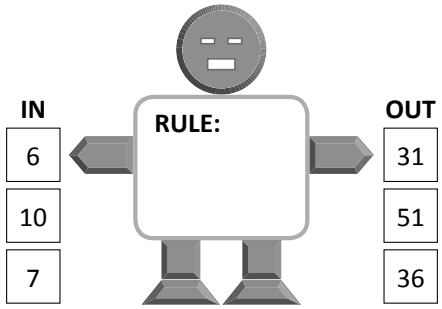

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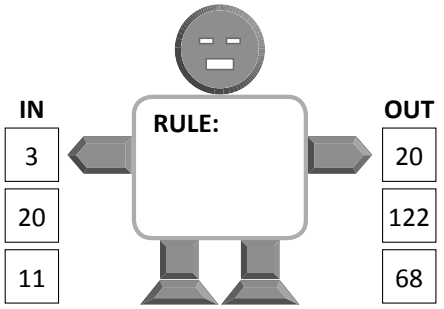
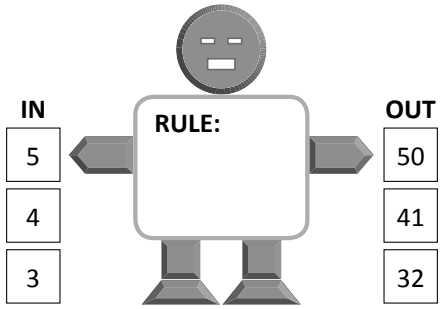
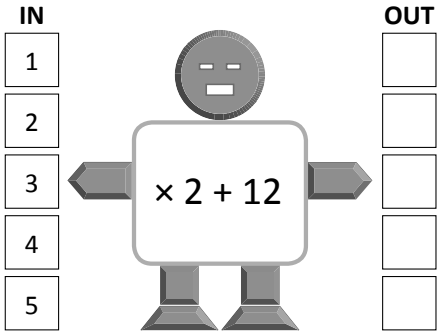


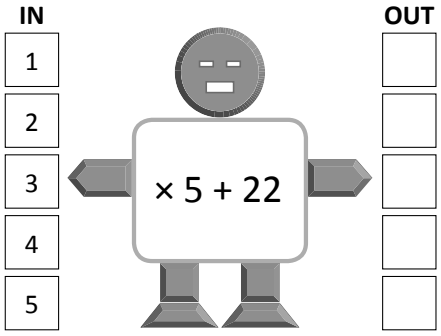
d



5 **Which function machine will win this game of bingo? Write the numbers that come out and colour each machine's numbers in a different colour. Check which machine has 3 numbers in a line in any direction.**

MATH x÷ BINGO +−				
27	16	45	12	17
42	32	22	18	23
47	68	FREE SPACE	18	29
15	20	37	15	32
14	30	43	16	35





Patterns and functions – function tables with addition and subtraction

The function machines showed us that when a number goes in, it comes out changed by the rule or the function. There are many function patterns in real life.

Look at this example:

At their Christmas fair, Middle Street Primary School charges $1.50 for a gift wrapping service. This table shows the total cost of each wrapped gift and shows the rule.

Cost of unwrapped gift	$7	$10	$15	$18
Cost of wrapped gift	$8.50	$11.50	$16.50	$19.50
Rule	Cost of unwrapped gift + $1.50 = Cost of wrapped gift			

1 **Complete the function table for the total cost of a day out at a fun park. You must pay an entry fee of $12 and purchase a wrist band for the amount of rides that you want to go on.**

Wrist band	5 rides for $20	6 rides for $25	7 rides for $30	8 rides for $35
Total admission				
Rule	Wrist band + $12 = Total cost			

2 **Complete the function table for the total cost of lunch at a school canteen. Students pay $2.40 for a sandwich and then choose what else they would like. Work out the total cost of lunch for each option.**

Lunch option	Drink: 80¢	Fruit: 95¢	Yoghurt: $1.10	Ice block: $1.50
Total cost of lunch				
Rule	Lunch option + $2.40 = Total cost of lunch			

3 **5F have fitness every Thursday afternoon for 30 minutes. Each week they complete a fitness activity and then play running games. Work out how much time is left for games after each activity.**

Activity	Skipping 10 minutes	Star jumps 12 minutes	Push ups 15 minutes	Sit ups 16 minutes
Time left for games				
Rule	30 minutes – length of time of activity = Time left for games			

Patterns and functions – function tables with multiplication

Let's look at more real life function tables, this time based on multiplication.

By working out the function, you can extend the pattern to find out unknowns.

For example:

A bakery makes 10 cupcakes an hour.

The rule to work out the number of cupcakes this bakery produces within a certain amount of time is:

Number of hours × 10 = Number of cupcakes

Hours	1	2	3	4	5	6	7	8
Cupcakes	10	20	30	40	50	60	70	80

How many cupcakes will it make in 1 day?

This table only goes up to 8 hours but we can use the function to answer this question:

24 hours × 10 cupcakes = 240 cupcakes

1 **Complete the function tables, write the rule and answer the question.**

a

A dry cleaner charges $2 to iron a shirt.								
Number of shirts	1	2	3	4	5	6	7	8
Cost	$2	$4	$6					
Write the rule for finding out the cost of ironing shirts when you know how many shirts:								
How much does it cost to have 12 shirts ironed?								

b

Monica and Anna have a lemonade stand outside their house. For every litre of lemonade they make 4 cups to sell.								
Litres	1	2	3	4	5	6	7	8
Cups	4	8						
Write the rule for finding out how many cups are needed when you know how many litres have been made:								
How many cups will be needed if they have enough to make 12 litres of lemonade?								

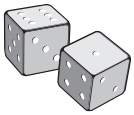
c

At a cinema, the candy is sold by weight. 1 scoop costs 50¢.								
Scoops of candy	1	2	3	4	5	6	7	8
Cost	50¢	$1						
Write the rule to find out the cost of the candy when you know how many scoops:								
How many scoops of candy can I get for $10?								

Rows and columns — apply

Getting ready

This is a game for 2 players. For this game you will need 2 dice, this page and 12 counters each, in 2 different colours. A calculator is optional.



What to do

Roll both dice, add them together and put this value in the function rule.

For example, if I roll 3 and 5, I add these and get 8. I put 8 into the first rule and get 8 × 7 – 3 = 53. I place one of my counters on 53.

If the answer is already taken, you lose a turn.

The winner is the player with the most counters in any row or column after 3 rounds of each function rule. (The numbers do not have to be next to each other, although you could play like that if you wanted a longer game.)

Function Rule 1	**Function Rule 2**	**Function Rule 3**
◈ × 7 – 3	6 × ⊙	(8 × ▣) – 5

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

What to do next

Change the object of the game. For example, the winner might be the person who has their counters on the most even numbers.

Pizza Pizzazz

solve

Getting ready

Pizza Pizzazz is the name of a pizza delivery company that you work for on the weekends. You drive all around town delivering hot and tasty pizzas in record time.

To encourage you to uphold the company guarantee of delivering pizzas in record time, your boss has given you a choice of bonus scheme.

What to do

Which scheme pays the best bonus?

Use the tables below to work out which payment system is best.

Payment System 1 For each pizza that you deliver, you will get $2.	
Number of pizzas	**Bonus**
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

Payment System 2 For each pizza that you deliver, your bonus will double, starting at 50¢.	
Number of pizzas	**Bonus**
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

Which bonus scheme would you choose and why?

What to do next

Can you think of when the other bonus scheme would be better?

Which bonus scheme do you think your boss would prefer you to choose?